ENTHALPY AND BOILING CROSS SECTION OF A LIQUID IN A HEATED TUBE

R. S. Kuznetskii

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An expression is derived for the enthalpy of an incompressible turbulent liquid flow in a heated tube. The location of the boiling section is found in general form.

We shall examine, in the hydraulic approximation, turbulent liquid flow in a straight cylindrical heated tube with a given linear heat flux density $q \geq 0$, assuming that the parameters of the liquid are averaged over the cross section and that heat entering from the outside is propagated instantaneously.

By boiling cross section we understand the cross section where the mean temperature of the liquid is equal to the saturation temperature, disregarding a certain section where, in actuality, boiling takes place close to the wall [1].

The moment θ, at which an element of liquid has the coordinate ξ, is related to that parameter by the differential equation

$$d\xi = w(\xi, \theta)\, d\theta. \tag{1}$$

If at a certain moment t the coordinate l of an element of liquid is assumed to be known, then the general solution of (1) will contain l and t as parameters:

$$\theta = \theta(\xi;\ l,\ t). \tag{2}$$

These determine the moment at which an element of liquid enters the tube:

$$\theta_0 = \theta(0;\ l,\ t) \equiv \theta_0(l,\ t). \tag{3}$$

The enthalpy of an element of liquid can be written in two forms:

$$i(l,\ t) = i(0,\ \theta_0) + \int_0^l \frac{q(\xi,\theta)}{G(\xi,\theta)}\, d\xi = i(0,\ \theta_0) + \frac{1}{S}\int_{\theta_0}^{t} \frac{q(\xi,\theta)}{\rho(\xi,\theta)}\, d\theta, \tag{4}$$

where θ_0 is the function (3) and in the first of the integrals θ is the function (2), while in the second ξ is the inverse of (2), and

$$\rho(\xi,\ \theta) = \rho\,[i(\xi,\ \theta),\ p(\xi,\ \theta)]. \tag{5}$$

We will restrict ourselves to the case of an incompressible fluid $\rho = \text{const}$. The velocity $w = w(t)$ must be specified at the entrance to the tube. In this case Eq. (1) is integrated in quadratures:

$$\xi = \int_{\theta_0}^{\theta} w(\theta)\, d\theta, \qquad l = \int_{\theta_0}^{t} w(\theta)\, d\theta. \tag{6}$$

The second of equalities (6) gives $\theta_0(l,\ t)$, after which the first gives $\xi(\theta; l,\ t)$ and (4) assumes the form

$$i(l,\ t) = i\,[0;\ \theta_0(l,\ t)] + \frac{1}{S\rho}\int_{\theta_0(l,t)}^{t} q\left[\int_{\theta_0(l,t)}^{\theta} w(\theta)\, d\theta,\ \theta\right] d\theta. \tag{7}$$

For steady-state heating constant along the tube $q = \text{const}$:

$$i(l,\ t) = i\,[0;\ \theta_0(l,\ t)] + \frac{q}{S\rho}\,[t - \theta_0(l,\ t)]. \tag{8}$$

For steady-state flow w = const:

$$i(l,\ t) = i\left(0,\ t - \frac{l}{w}\right) + \frac{1}{S\rho}\int_0^{\frac{l}{w}} q(l - w\eta,\ t - \eta)\,d\eta. \qquad (9)$$

When q = const and w = const simultaneously,

$$i(l,\ t) = i\left(0,\ t - \frac{l}{w}\right) + \frac{q}{G}\,l, \qquad (10)$$

and, finally, in the absolutely stationary case when the liquid entering the tube also has constant enthalpy,

$$i(l) = i(0) + \frac{q}{G}\,l. \qquad (11)$$

The boiling cross section is determined by the condition

$$i(l,\ t) = i'[p\ (l,\ t)], \qquad (12)$$

where $i'(p)$ is a known function [2].

The pressure and velocity of the liquid in the tube are given by the system [3]

$$\begin{gathered} -\frac{1}{\rho}\ \frac{\partial p}{\partial l} = w\frac{\partial w}{\partial l} + \frac{\partial w}{\partial t} + \frac{\lambda}{2d}w^2 + g, \\ \frac{\partial}{\partial l}\ (\rho w) + \frac{\partial \rho}{\partial t} = 0, \quad \rho = \rho(i,\ p) \end{gathered} \qquad (13)$$

and the corresponding boundary and initial conditions.

In particular, with ρ = const

$$p(l,\ t) = p(0,\ t) - \rho l\left(\frac{\partial w}{\partial t} + \frac{\lambda}{2d}\ w^2 + g\right), \qquad (14)$$

where p(0, t), like w(t), is specified at the entrance to the tube. With w = const

$$p(l) = p(0) - \rho l\left(\frac{\lambda}{2d}\,w^2 + g\right). \qquad (15)$$

Now condition (12), after substitution of (7) and (14), permits determination of the boiling cross section of the liquid, which in the general case depends on time, $l = l(t)$.

NOTATION

l, ξ – coordinates along tube; t, θ – time; θ_0 – moment at which element of liquid enters tube; G – mass flow rate; w – velocity; ρ – density; p – pressure; q – linear heat flux density; i' – enthalpy at saturation line; d and S – inside diameter and cross section of tube; g – projection of vector of gravitational acceleration in direction of tube; λ – coefficient of friction of tube.

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"Giprostal" Institute,
Khar'kov